Exercise 1.1

1 Find the values of the letters p, q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \frac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \frac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\frac{60}{x+4} = 12$ (b) $\frac{35}{2x-3} = 5$ (c) $\frac{20}{6-x} = \frac{1}{2}$

3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab\cos C$.

4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x+2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations.

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations.

(a) $\frac{a}{b}(cx+d) = x+2$ (b) $\frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$

7 Simplify the following as far as possible.

(a) $\frac{a+a+a+a+a}{5}$ (b) $\frac{b+b+b+b}{b}$

(c) $\frac{c\times c\times c\times c\times c}{c}$ (d) $\frac{d\times d\times d\times d}{4}$

Exercise 1.2

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7}\times 5$ (b) $\frac{5}{12}\times 3$ (c) $\frac{7}{9}\times 2$ (d) $\frac{4}{15}\times 3$

(e) $\frac{8}{11}\div 4$ (f) $\frac{8}{11}\div 3$ (g) $\frac{6}{7}\div 3$ (h) $\frac{6}{7}\div 5$

(i) $\frac{3x}{y}\times x$ (j) $\frac{3x}{y^2}\times y$ (k) $\frac{5x^3}{4y}\div x$ (l) $\frac{5x^2}{6y}\div y$

(m) $\frac{5x^3}{2y}\times 3x$ (n) $\frac{3y^4}{4x^2z}\times 2x$ (o) $\frac{6x^2y^3}{5z}\div 2xy$ (p) $\frac{5a^2}{6x^3z^2}\div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V = \frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u+v) = tx$ (d) $W = \frac{2}{3}\pi x^2 h$

3 Simplify the following compound fractions.

(a) $\dfrac{\frac{1}{x}+1}{\frac{1}{x}+3}$ (b) $\dfrac{\frac{2}{x}+1}{\frac{3}{x}-1}$ (c) $\dfrac{\frac{1}{x+1}+2}{\frac{1}{x+1}+1}$

4 Write as single fractions.

(a) $\frac{2}{x-1}+\frac{1}{x+3}$ (b) $\frac{2}{x-3}-\frac{1}{x+2}$ (c) $\frac{1}{2x-1}-\frac{1}{3x+2}$ (d) $\frac{3}{x+2}+1$

(e) $2-\frac{1}{x-1}$ (f) $\frac{2x}{x+1}-3$ (g) $\frac{3}{4(2x-1)}-\frac{1}{4x^2-1}$

5 Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}}+\sqrt{x}$ (b) $\frac{2x}{\sqrt{x+3}}+\sqrt{x+3}$ (c) $\frac{x}{\sqrt[3]{x-2}}+\sqrt[3]{(x-2)^2}$

6 Write the following in the form $1+\frac{a}{x+b}$.

(a) $\frac{x+1}{x-5}$ (b) $\frac{x+3}{x+1}$ (c) $\frac{x+2}{x+5}$ (d) $\frac{x-6}{x-2}$

7 Write the following equations without fractions. (A, B etc. are constants that remain in your answers.)

(a) $\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

(b) $\frac{x+2}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$

(c) $\frac{2}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$

(d) $\frac{1}{(x-2)^2(x+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+1}$

(e) $\frac{1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

Exercise 1.3

1 Write without brackets.

(a) $(x+5)^2$ (b) $(x-4)^2$ (c) $(2x+1)^2$

(d) $(3x-2)^2$ (e) $(x+2)(x-2)$ (f) $(3x+4)(3x-4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x+3)^2 + (y+4)^2 = (x-2)^2 + (y-1)^2$

(b) $(x+5)^2 + (y+2)^2 = (x-5)^2 + (y-2)^2$

(c) $(2x+1)^2 + (y-3)^2 = (2x+3)^2 + (y+1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2+4}$ (b) $\sqrt{x^2-4x+4}$ (c) $\sqrt{x^2-1}$

(d) $\sqrt{x^2+9x}$ (e) $\sqrt{x^2-y^2}$ (f) $\sqrt{x^2+2xy+y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

5 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

6 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$ (b) $9x^2 - 12x - 1$ (c) $16x^2 + 40x + 22$

7 Factorise as fully as possible.

(a) $x^2 - 25$ (b) $4x^2 - 36$ (c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$ (e) $3x^2 - 5x + 2$ (f) $6x^2 - 5x - 6$

(g) $8x^2 - 2x - 15$

8 Multiply out and simplify.

(a) $\left(x+\frac{1}{x}\right)^2$ (b) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ (c) $\left(x+\frac{2}{x}\right)\left(x-\frac{3}{x}\right)$

Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2.$

(c) $\dfrac{4+6x}{2}$ (d) $\dfrac{4\times 6x}{2}$ (e) $\dfrac{3x+xy}{x}$

(f) $\dfrac{3x\times xy}{x}$ (g) $\dfrac{4x+10y}{8x+6y}$ (h) $\dfrac{3x-6y}{9x-3y}$

(i) $\dfrac{4x+9y}{2x+3y}$ (j) $\dfrac{4x+6y}{6x+9y}$ (k) $\dfrac{5xy+6y^2}{10x+12y}$

(l) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ (m) $\dfrac{x-3}{3-x}$ (n) $\dfrac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b} = \dfrac{py}{qz}$ (b) $\dfrac{3\pi ax}{b} = \dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\dfrac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c) $\frac{1}{2}k(k+1) + (k+1)$ (d) $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

5 Simplify as far as possible.

(a) $\dfrac{x^2+6x+8}{x^2-x-6}$

(b) $\dfrac{3x^2-2x-8}{x^2-4}$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3}$

(d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

(e) $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}}-\sqrt{x^2+1}}{x^2}$

(f) $\dfrac{-\dfrac{x}{2\sqrt{1-x}}+\sqrt{1-x}}{x^2}$

(g) $\dfrac{\dfrac{\sqrt{x}}{2\sqrt{1+x}}-\dfrac{\sqrt{1+x}}{2\sqrt{x}}}{x}$

(h) $\dfrac{\sqrt[3]{1+x}-\dfrac{x}{3\sqrt[3]{(1+x)^2}}}{\sqrt[3]{1+x}}$

Exercise 1.5

Solve the following simultaneous equations.

1 $x^2+xy=12$

$3x+y=10$

2 $x^2-4x+y^2=21$

$y=3x-21$

3 $x^2+xy+y^2=1$

$x+2y=-1$

4 $x^2-2xy+y^2=1$

$y=2x$

5 $c^2+d^2=5$

$3c+4d=2$

6 $x+2y=15$

$xy=28$

7 $2x^2+3xy+y^2=6$

$3x+4y=1$

8 $2x^2+4xy+6y^2=4$

$2x+3y=1$

9 $4x^2 + y^2 = 17$

$2x + y = 5$

10 $2x^2 - 3xy + y^2 = 0$

$x + y = 9$

11 $x^2 + 3xy + 5y^2 = 15$

$x - y = 1$

12 $xy + x^2 + y^2 = 7$

$x - 3y = 5$

13 $x^2 + 3xy + 5y^2 = 5$

$x - 2y = 1$

14 $4x^2 - 4xy - 3y^2 = 20$

$2x - 3y = 10$

15 $x^2 - y^2 = 11$

$x - y = 11$

16 $\frac{12}{x} + \frac{1}{y} = 3$

$x + y = 7$

Exercise 1.6

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x + \frac{1}{x}\right)$ (b) $\frac{x^4 + 1}{x^2}$ (c) $x^{-5}\left(x + \frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

7 Simplify $\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{100}+\sqrt{99}}$.

Exercise 3.1

1 Rearrange the following in the form $ax + by + c = 0$ or $ax + by = c$ as convenient, where a, b and c are integers and $a > 0$.

(a) $y = 3x - 2$ (b) $y = \frac{1}{2}x + 3$

(c) $y = -\frac{3}{4}x + 3$ (d) $y = \frac{7}{2}x - \frac{5}{4}$

(e) $y = -\frac{2}{3}x + \frac{3}{4}$ (f) $y = \frac{4}{7}x - \frac{2}{3}$

2 Rearrange the following in the form $y = mx + c$. Hence find the gradient and the y-intercept of each line.

(a) $2x + y = 8$ (b) $4x - y + 9 = 0$

(c) $x + 5y = 10$ (d) $x - 3y = 15$

(e) $2x + 3y + 12 = 0$ (f) $5x - 2y = 20$

(g) $3x + 5y = 17$ (h) $7x - 4y + 18 = 0$

3 Sketch the following lines. Show on your sketches the coordinates of the intercepts of each line with the x-axis and with the y-axis.

(a) $2x + y = 8$ (b) $x + 5y = 10$

(c) $2x + 3y = 12$ (d) $3x + 5y = 30$

(e) $3x - 2y = 12$ (f) $4x + 5y + 20 = 0$